Risk Modelling

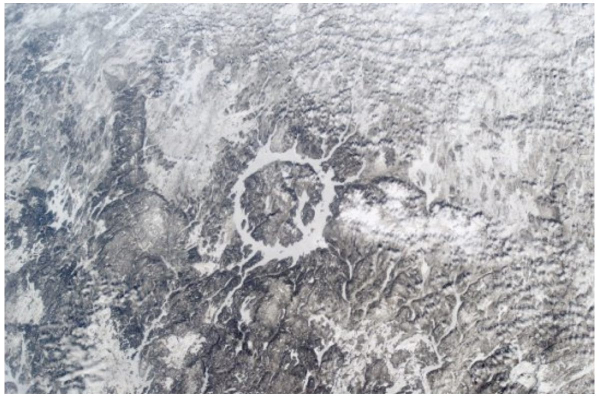
Topics

Part I :Risk definitions and modeling in economics

Part II : Modelisation of risk dependance structure.


50 milliards de soleils
à 240 000 000 yl
dans NGC 1260
SN 2006gy


Eta Carinae à 7000 al en fin de vie
explosera bientôt (ou a déjà explosé!)




G.B. 2007


St Helen

Landslides

TSUNAMIS

Adverse Selection ?

Or Moral Hazard ?

BLAZES : Moral Hazard or Adverse Selection ?

La Provence

LUNDI 19 MAI 2008 | AIX-PAYS D'AIX | www.laprovence.com / 0,90€

BALL

/ PHOTO BRUNO SQUILLARD

2008 :
Mandanda

Les maisons hors la loi dans les zones à risques

Menaces Inondations, feux de forêt, avalanches, séismes... les risques naturels sont nombreux dans notre région.

Plans L'État a ainsi identifié huit risques majeurs et chargé les préfectures d'élaborer des plans de prévention.

Mesures Outre le refus des permis de construire, expropriations et démolitions sont désormais appliquées. ▸ page 24

Zones à risques : les maisons pourront être détruites

Thierry Noir et nos agences
tnoir@laprovence-presse.fr

C'est une tendance lourde de notre société : la France du XXI[e] siècle veut tendre vers le "risque zéro". Dans notre région, les risques de catastrophes naturelles sont grands : qu'on se souvienne des 37 morts de Vaison-la-Romaine, après les crues torrentielles de l'Ouvèze, en 1992, ou des 8 000 sinistrés du pays d'Arles, quand, en 2003, le Rhône est sorti de son lit !

Les Alpes-de-Haute-Provence connaissent aussi des risques d'avalanche et de feux de forêt, alors que les Bouches-du-Rhône vivent avec les risques industriels des usines classées Seveso.

Pour répondre à l'évolution de la société, l'État a identifié huit risques majeurs. Chaque préfecture doit élaborer des plans de prévention de ces risques. Et en tirer les conséquences. Il ne s'agit même plus uniquement d'interdire la construction future de maisons dans des zones dorénavant jugées dangereuses

▸ Entre les risques de feux de forêt, d'avalanches et d'inondations, la région Paca possède de nombreuses zones dangereuses. En 2003, une crue du Rhône avait ainsi sinistré le quartier du Trébon, à Arles, comme le montre cette vue aérienne. / PHOTO ARCHIVES JÉRÔME REY

(*lire ci-dessous*). L'État va me jusqu'à racheter, pou démolir, les habitations autres implantations hu nes dans des endroits qués.

Ces plans, dont l'élab tion est longue, ne sont encore en vigueur. Mais jà, un certain nombre de rections départementale l'équipement les appliq "par anticipation", not ment dans le Var, et refu des permis de construir attaquent devant les tr naux ceux déjà signés.

Reste qu'il y a, par ex ple, des gens qui ont trav toute leur vie pour trans tre leur maison à leurs fants. Or celle-ci n'a plu valeur, car en cas d'ince de forêt, elle ne pourrait être reconstruite. D'au ont acheté des terra constructibles… sur lesq ils ne peuvent pas bâtir.

Si les intentions de l' sont louables, les indemn tions liées à l'applicatio la loi pourraient utilem être revues. ■

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Health Risks

SCIENTIFIC VS STATISTICAL KNOWLEDGE

e.g. : Risk of Aneurysm Rupture

- Statistical knowledge

(e.g. Logit-Probit Regressions after identifying the risk factors)

- Scientific knowledge applied to comprehensive clinical exams specific to the case.

- Experts' judgments (physicians or Artery specialists)

Q : Why do insurance companies do not propose medical insurance for senior people ? R: The risk is assessed by people according to their knowledge

- for younger people : statistical (common) knowledge dominates private knowledge

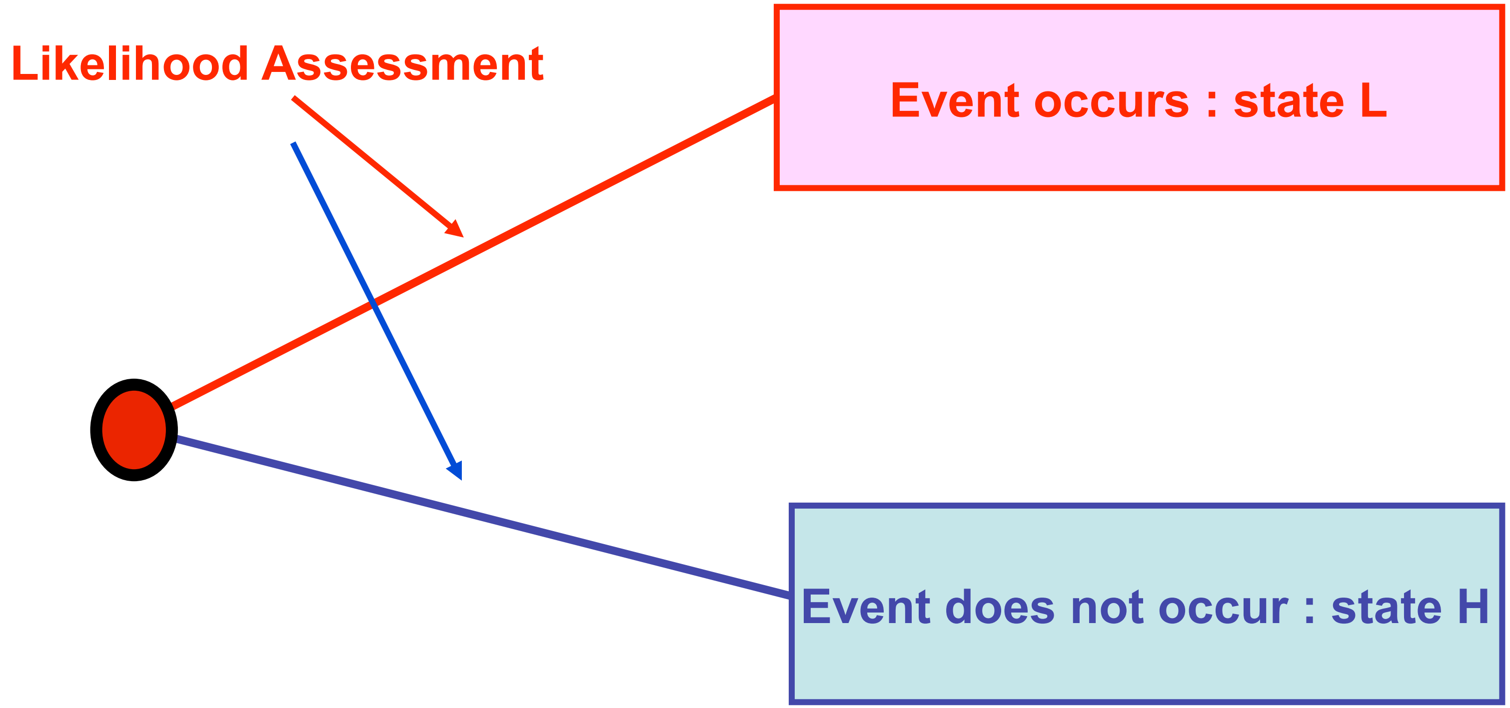
- for older people : personal records and private knowledge dominates statistical knowledge

This leads to adverse selection problems.

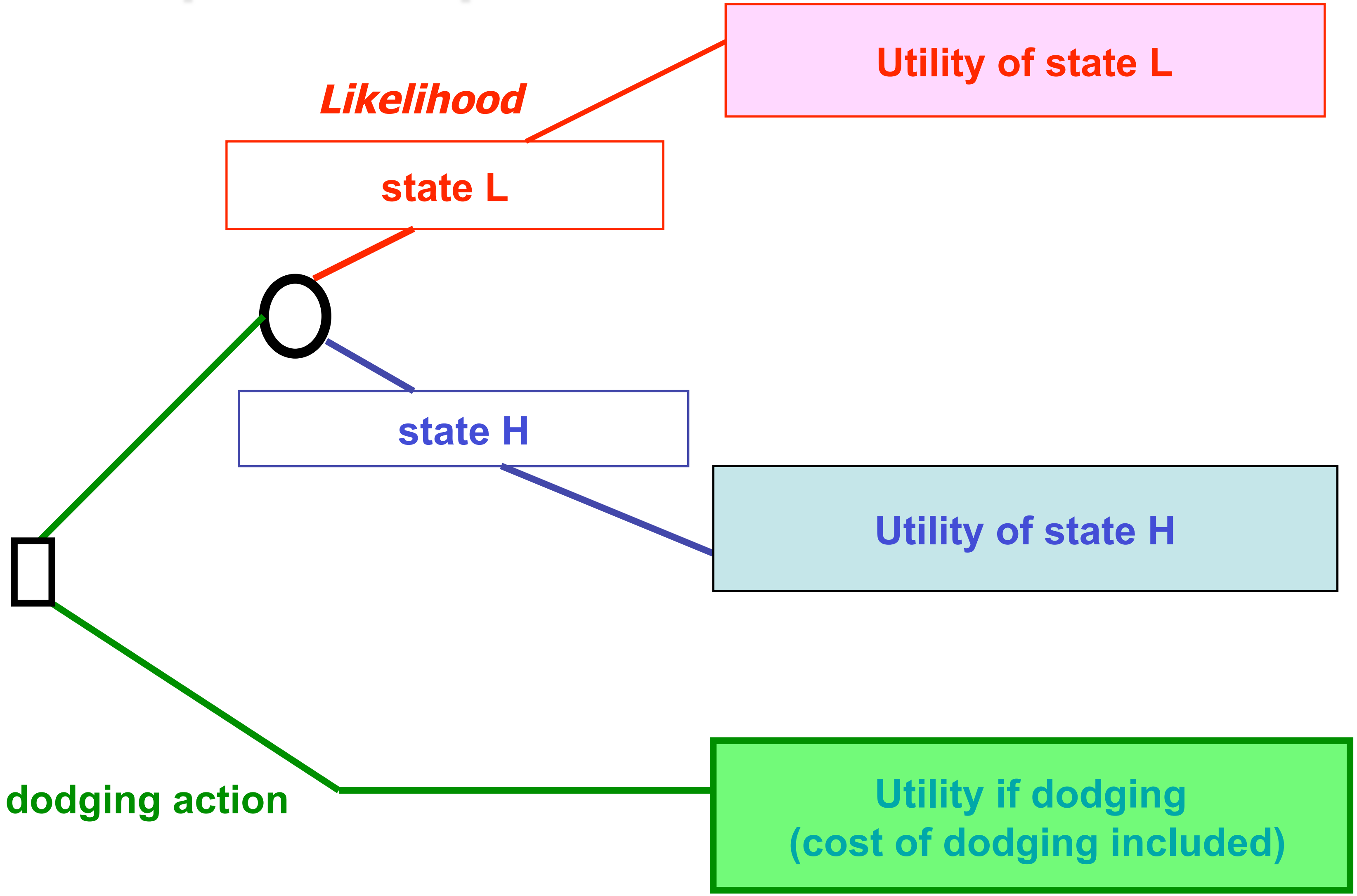
Risk and Decisions

Modelling Uncertainty and Decisions under Uncertainty

Uncertain Events



Simplified Representation of Risk Decision



RISK = MISSING INFORMATION

There is risk because one can choose the bad action

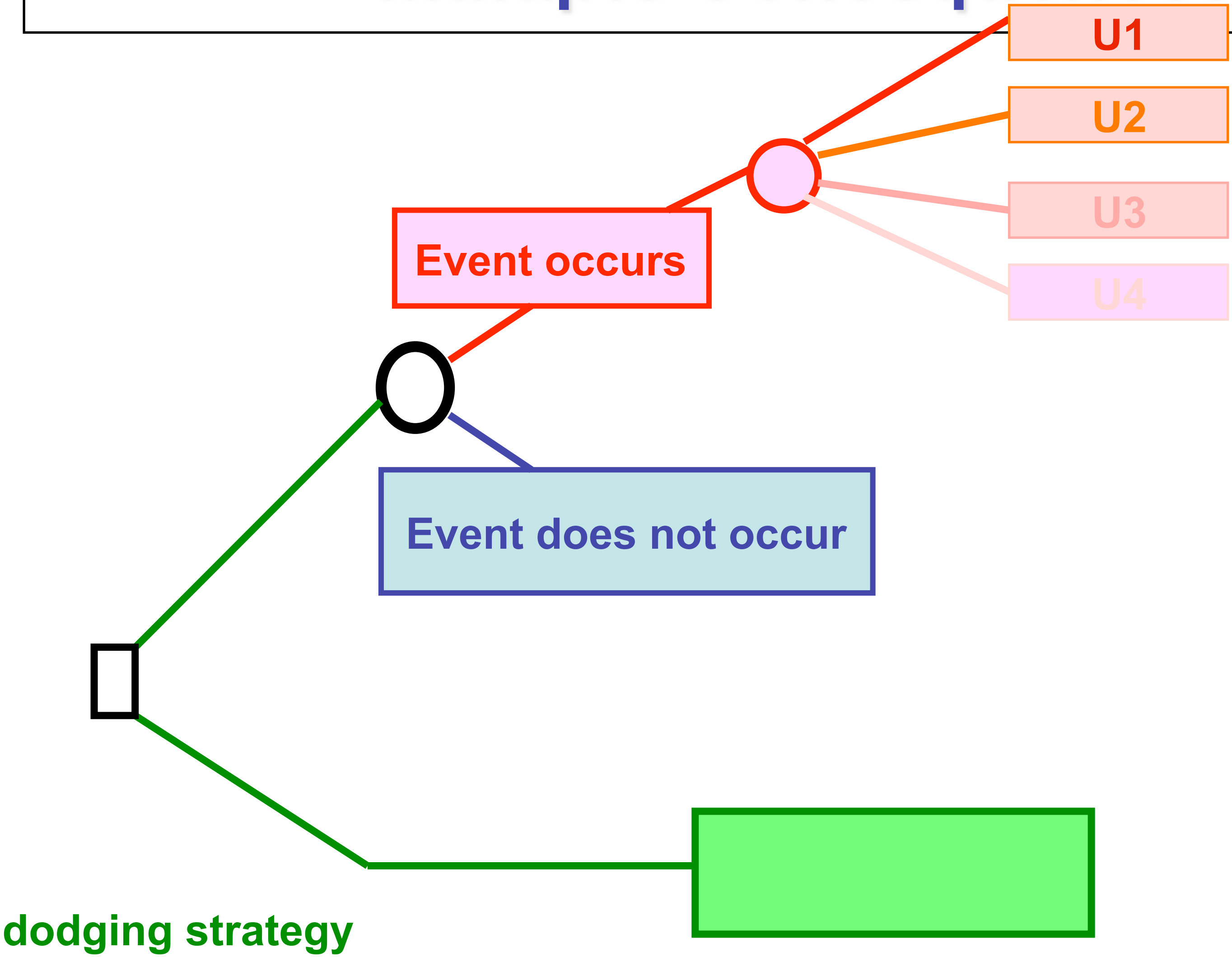
- When information is complete, there is no more risk

- When there is no possible dodging action, there is no risk.

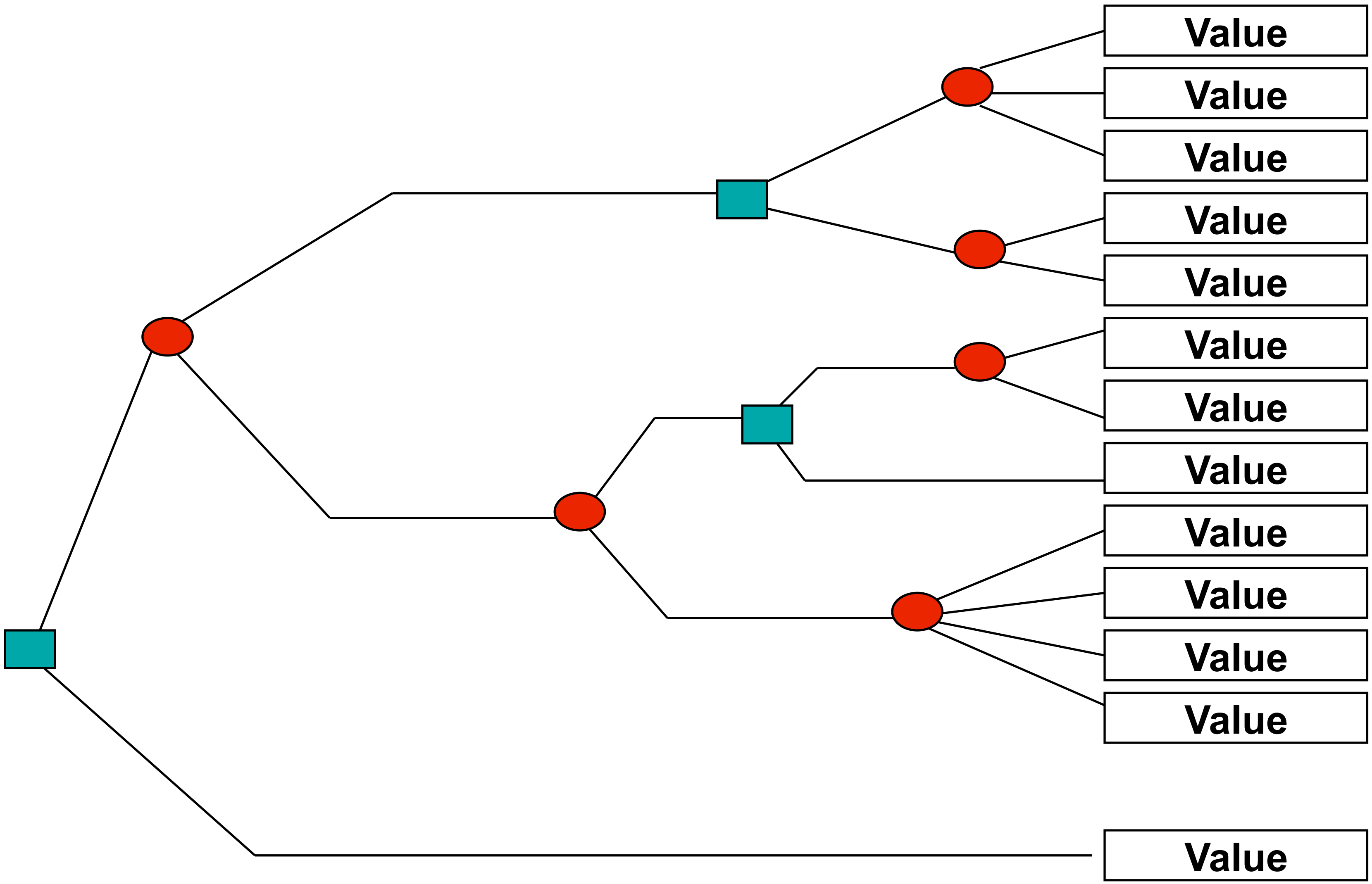
Pb : information is unequally shared by those who can control the dodging actions.

(Often those who know best are not those who act)

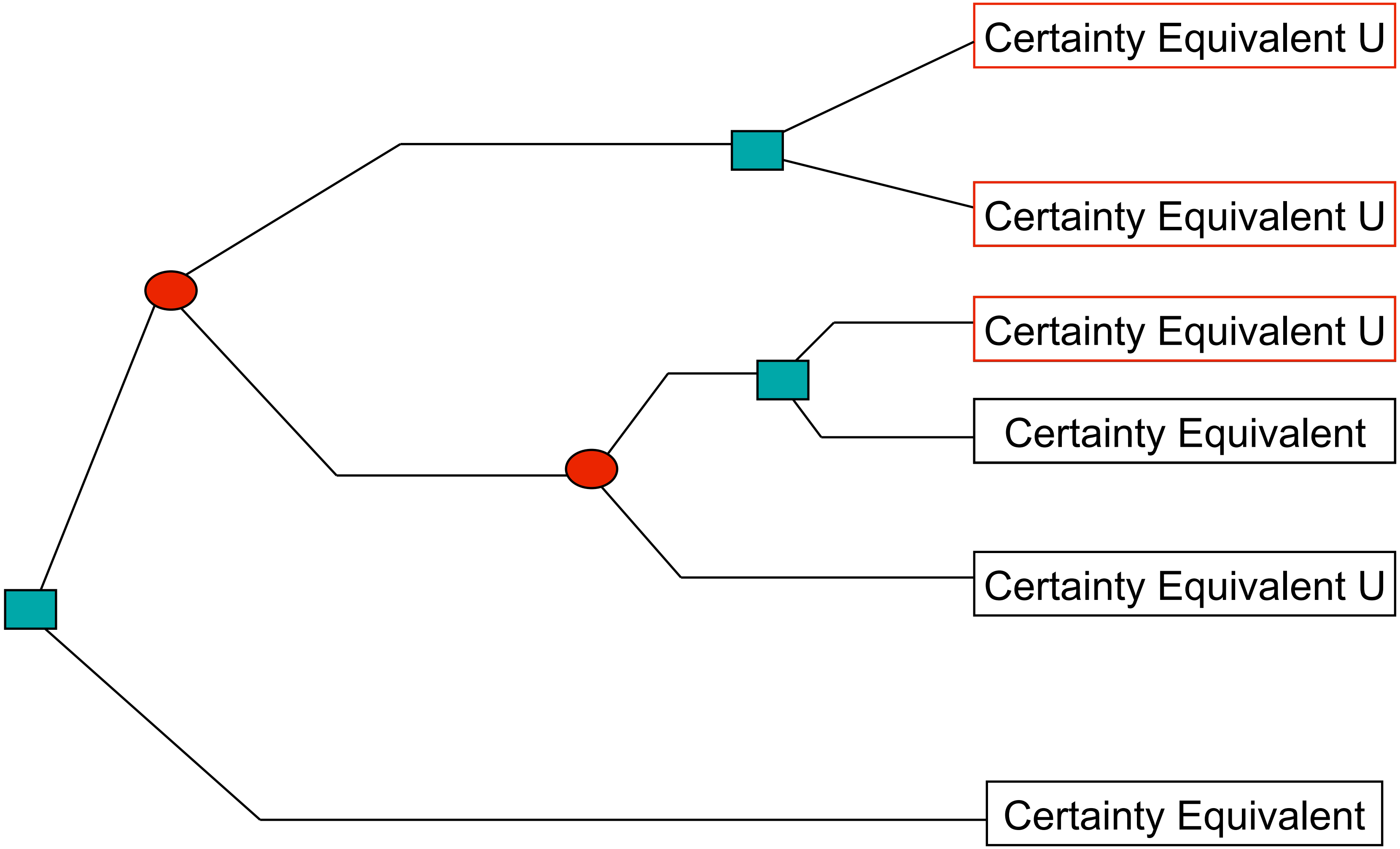
Multiple Consequences



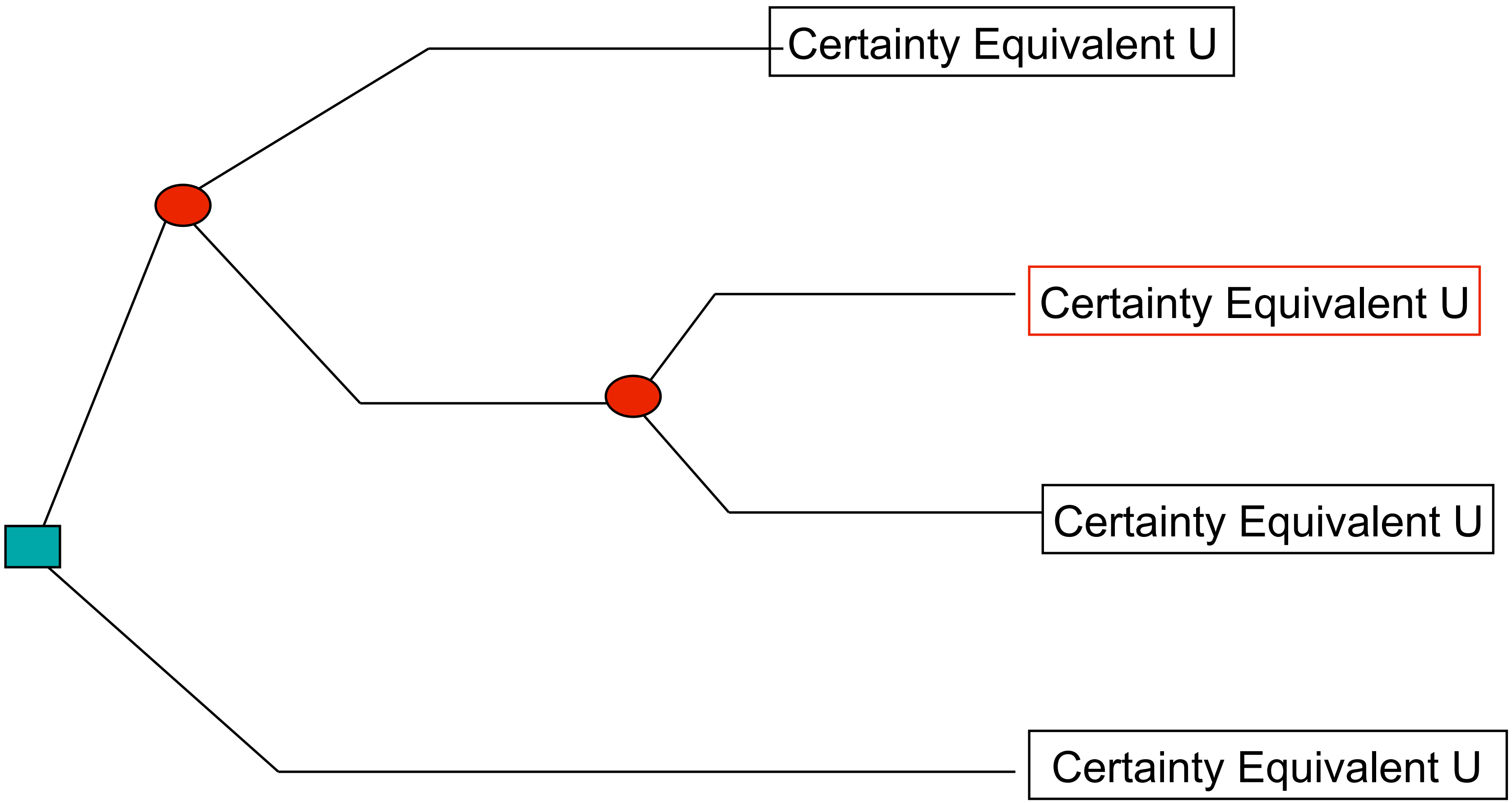
Decision Tree



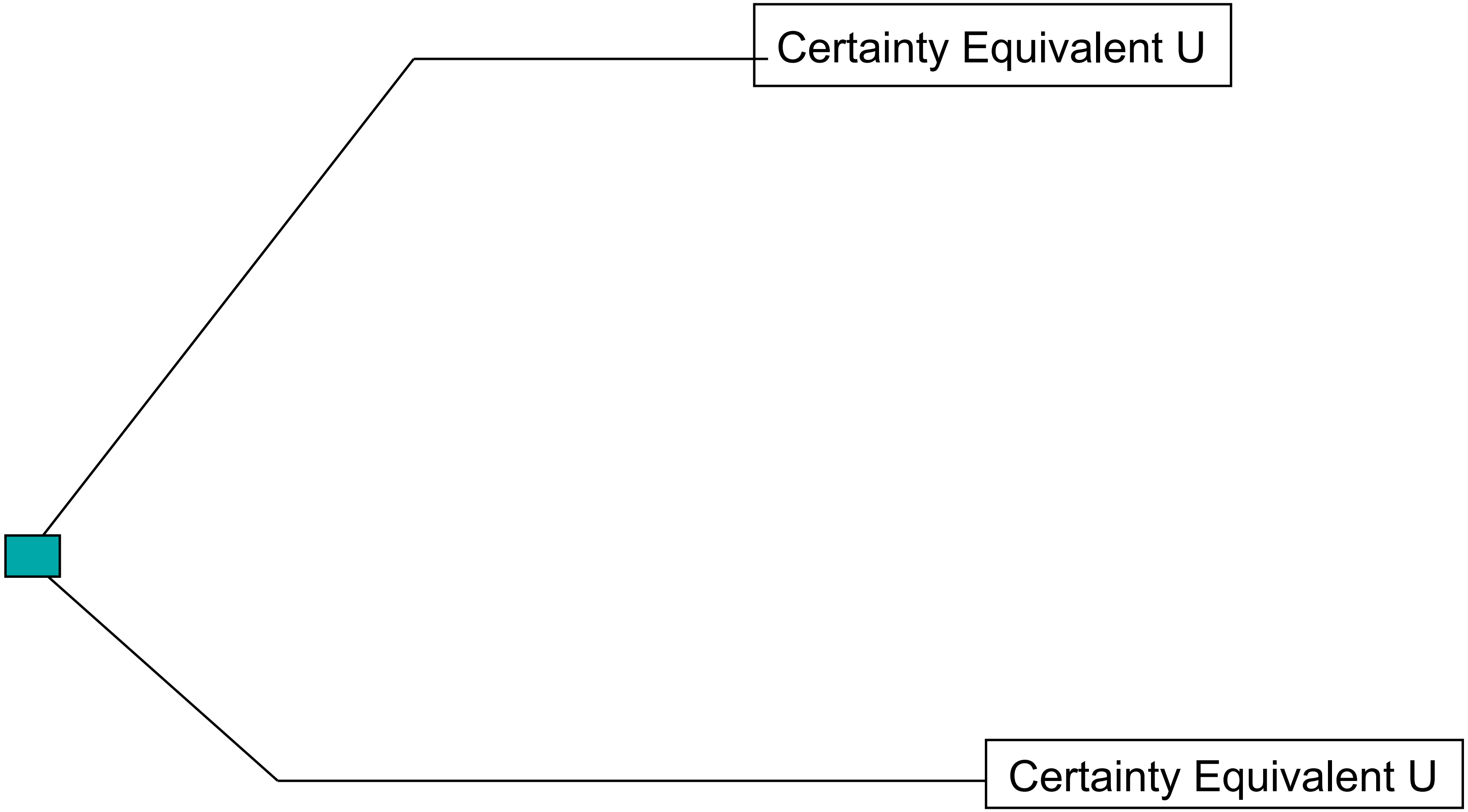
Decision Tree: Backward Induction



Decision Tree: Backward Induction II



Decision Tree: Backward Induction III



How to Value Uncertain Prospects?

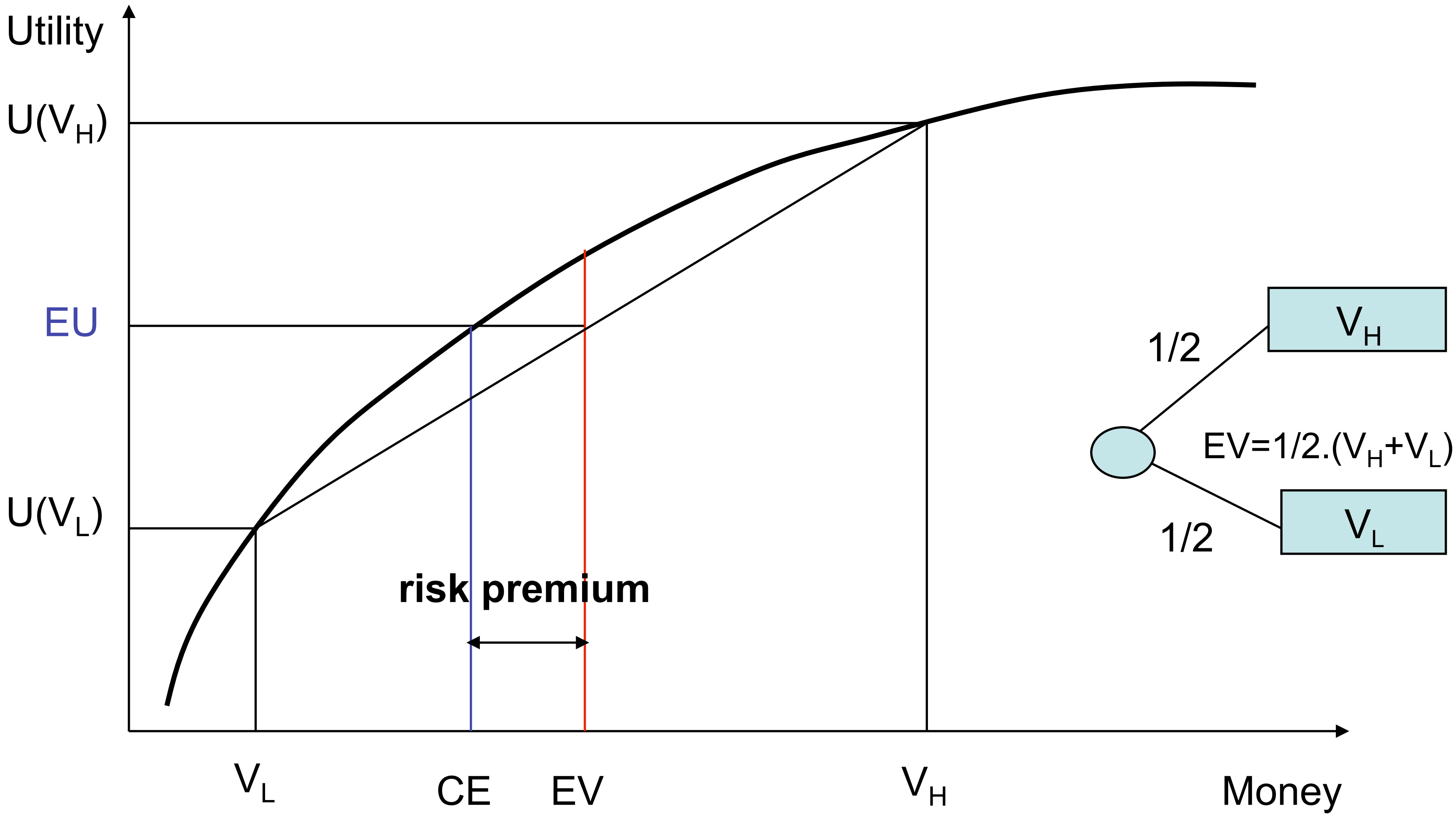
Problem 1: Likelihood : probability, capacities, scenarii (os), fuzzy sets, rankings...

Problem 2: independence "likelihood - valuation" ?

Problem 3: tractability (linear algebra, differential calculus, probability theory)

The Old Bernouilli Model

$U[pV_H+(1-p)V_l] > pU(V_H) +(1-p)U(V_L)$ (Jensen's inequality)



Risk Aversion

Most empirical studies in finance, insurance and economics exhibit the existence of RISK PREMIA.

They are evidence of a widespread human attitude toward risk, termed *"risk aversion"* and measured by different direct (e.g. Arrow-Pratt measures) or indirect artifacts

Bernouilli's Followers and Critics

<u>Founding fathers;</u>

- John Von Neuman
- Leonard Savage
- Jacob Marschak
- Kenneth Arrow & Gérard Debreu : a way to avoid
- Maurice Allais : sharp criticism of the founding fathers.
- Harry Markowitz, William Sharpe : risk in finance
- Amos Tverski & Daniel Kahneman : the psychologist
- Mark Machina the refounders
- David Schmeidler
- Itzhak Gilboa

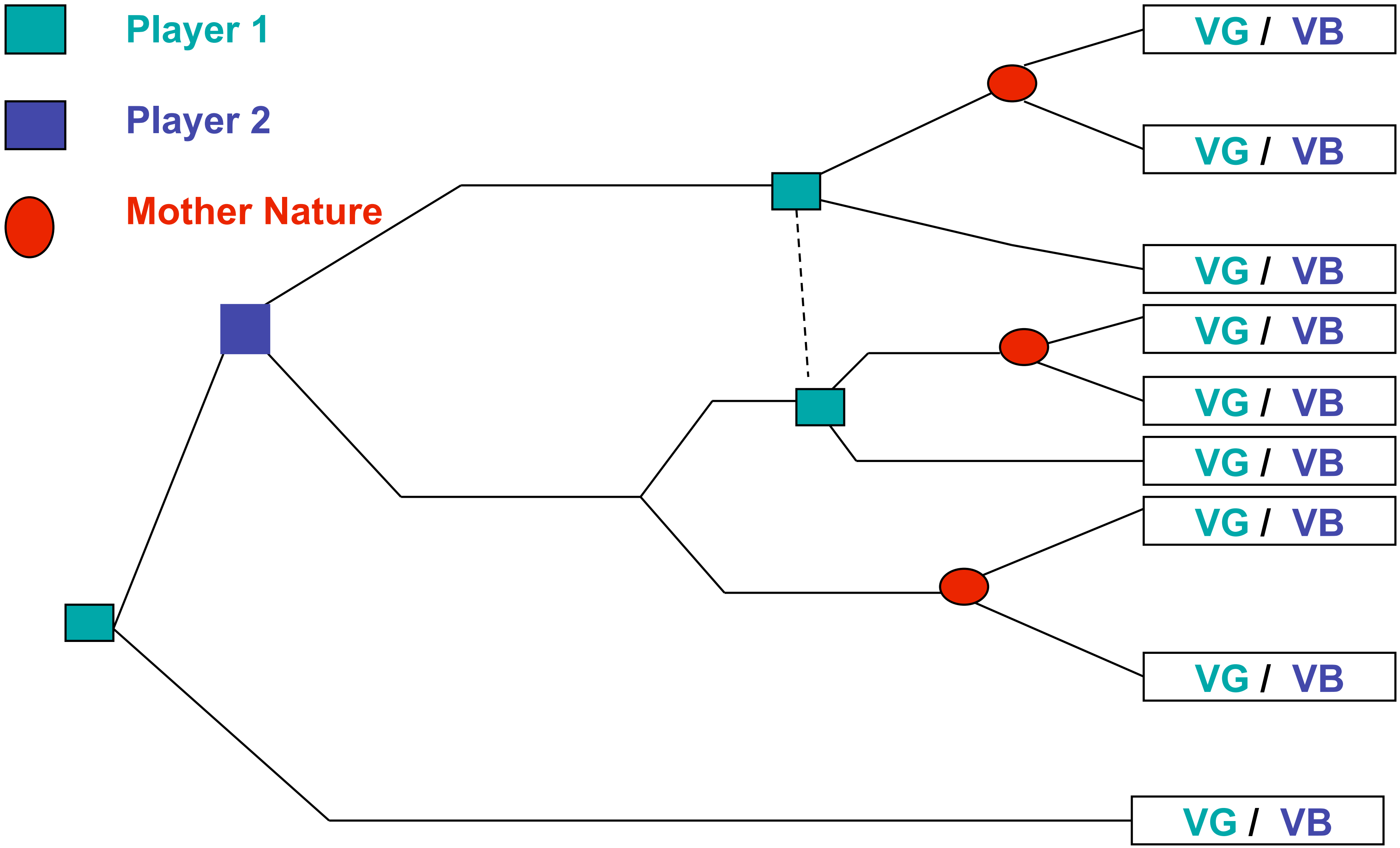
Source of Information for Assessing Likelihood

- Statistical knowledge about the risk occurence
- Scientific (specific) knowledge about the driving risk factor
- Experts' Judgments

Each risk assesment is a mix of the three sources.

- If scientific investigation is comprehensive enough, no need for statistical knowledge.
- When both statistical and scientific knowledge are weak, expert opinions dominate
- Expert judgments is expression of "tacit knowledge

Interactive Risks : Game Theory



Moral Hazard and Adverse Selection

Moral Hazard: risk for a given agent is induced by the action of another agents without possibility of monitoring.

Ex : car insurance : Risk for the insurer depends on traffic hazard risk **and** risk of misconduct of the driver who does not suffer consequences due to insurance coverage.

Adverse Selection: due to *asymmetry of information* an agent who seek to trade with low risk partners attracts high risk partners

Typical example : the Akerlof Model

Expert Opinions

Opinion expressions:

- Comparative Likelihood : binary relationships
- Likelihood rankings : complete preorders
- Fuzzy sets
- Choquet capacities
- Probabilities
- scenarii (ios)

Pb : Mitigate cognitive biaises.

Risk Analysis and Epistemic Logic

- Modelling information distributed among many agents
- What is "common knowledge" (mathematical difficulties : e.g. The e-mail paradox)
- Public knowledge is different than common knowledge.
- Level of relative knowledge
- What is "disagreement" ?

Aggregation of Expert Opinions

-Logical difficulties :

ex. aggregating probabilities

- Agreeing to disagree ? (R. Aumann)

- Cognitive and psychological difficulties

Ex: group influences on risk aversion

- Bayesian approaches

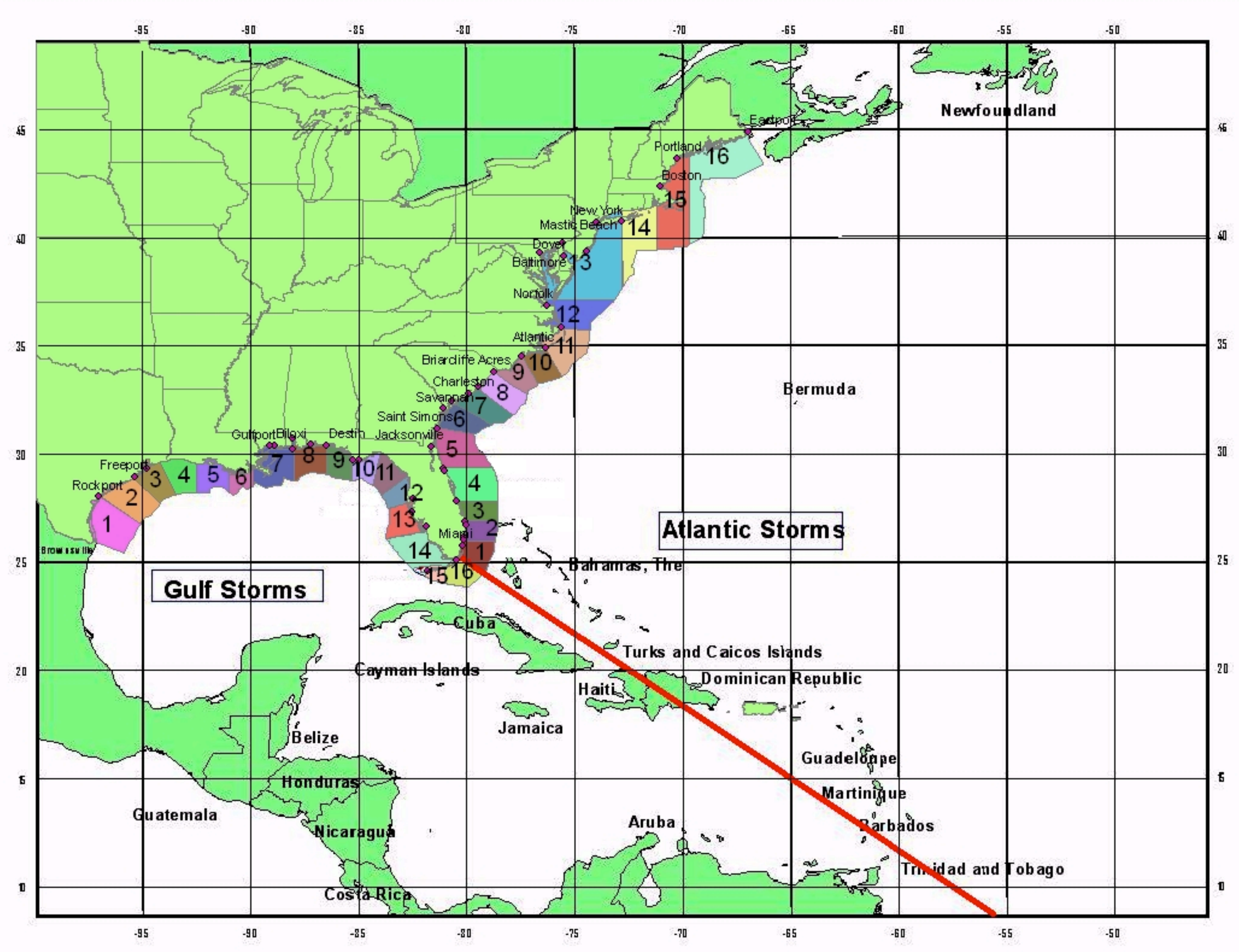
- Aggregation via market system

Aggregation by Market Systems: Hurricane Futures

Can a financial market predict

hurricane landfalls

better than expert risk models ?


Gulf Storms
Atlantic Storms
Newfoundland
Eastport
Portland
Boston
New York
Mastic Beach
Dover
Baltimore
Norfolk
Atlantic
Briarcliffe Acres
Charleston
Savannah
Saint Simons
Jacksonville
Miami
Gulfport
Biloxi
Destin
Freeport
Rockport
Brownsville
Bermuda
Bahamas, The
Cuba
Turks and Caicos Islands
Cayman Islands
Haiti
Dominican Republic
Jamaica
Belize
Guadeloupe
Honduras
Martinique
Guatemala
Aruba
Barbados
Nicaragua
Trinidad and Tobago
Costa Rica
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-90
-85
-80
-75
-70
-65
-60
-55
-50
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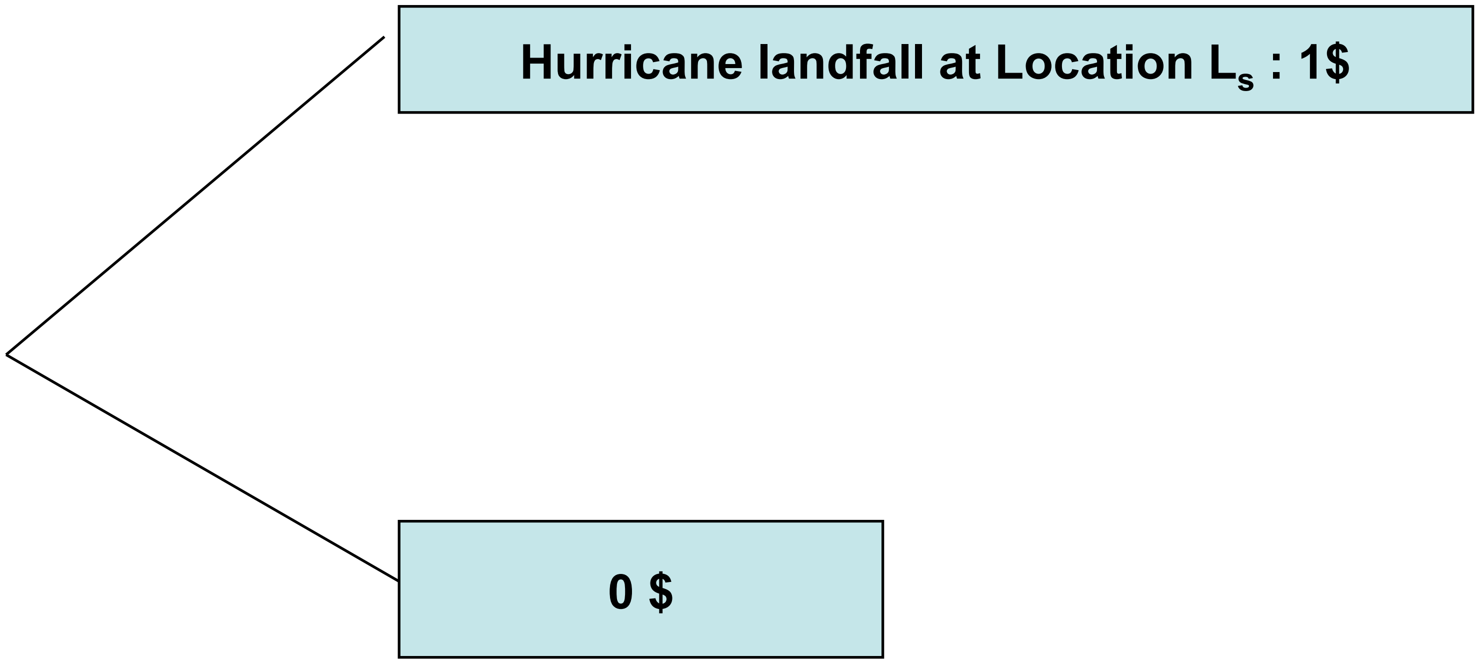
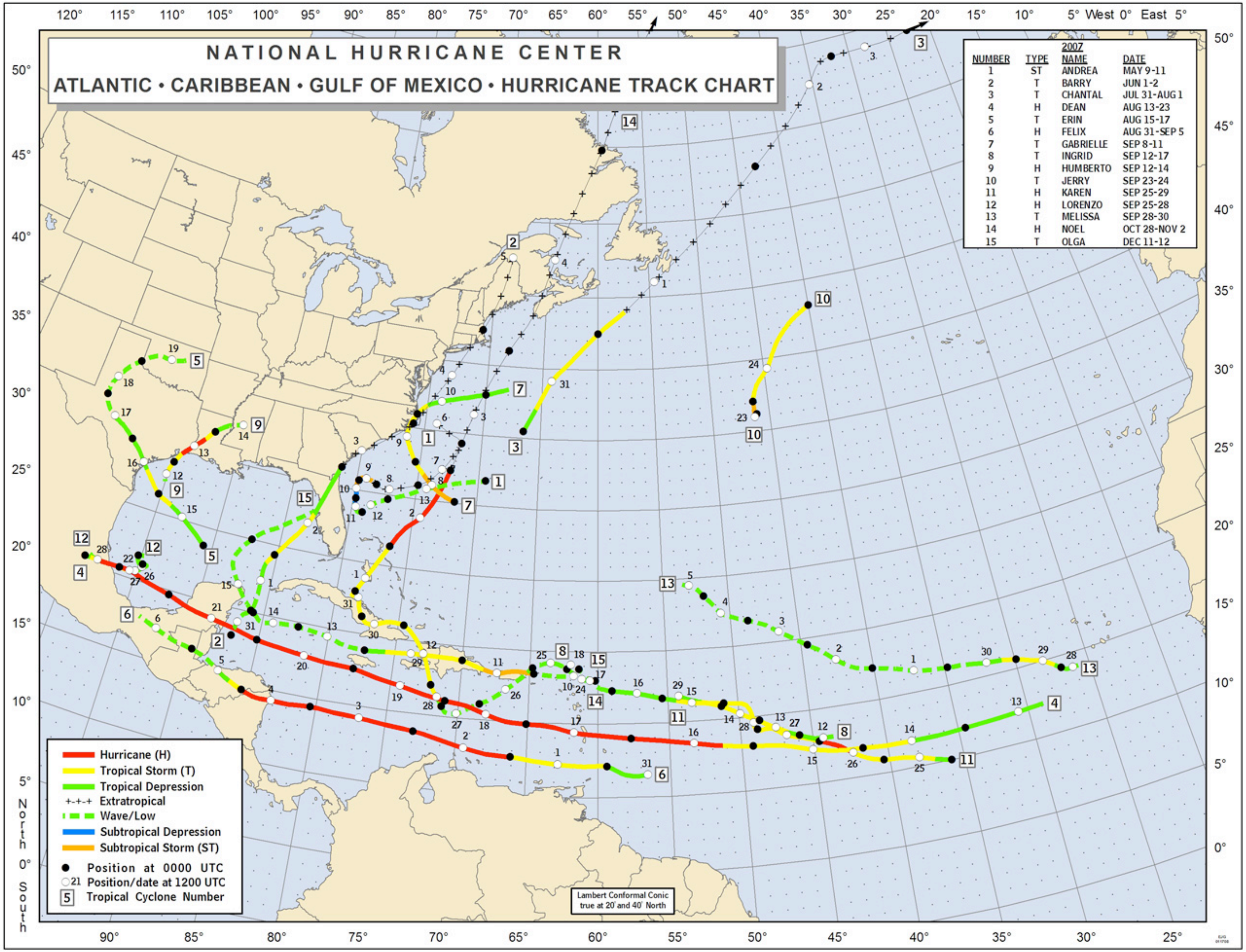
10

Liquidation (Final) Values of Contract L_s

1 contract for each location s s = 1, S (S can be up to 17)



Bundle of S contracts <u>Exhaustive and Exclusive</u>
<u>(One will necessarily end up with 1 and all other with 0)</u>
Initial bundle purchasing price 1$: fair game


NATIONAL HURRICANE CENTER
ATLANTIC • CARIBBEAN • GULF OF MEXICO • HURRICANE TRACK CHART
2007
NUMBER TYPE NAME DATE
1 ST ANDREA MAY 9-11
2 T BARRY JUN 1-2
3 T CHANTAL JUL 31-AUG 1
4 H DEAN AUG 13-23
5 T ERIN AUG 15-17
6 H FELIX AUG 31-SEP 5
7 T GABRIELLE SEP 8-11
8 T INGRID SEP 12-17
9 H HUMBERTO SEP 12-14
10 T JERRY SEP 23-24
11 H KAREN SEP 25-29
12 H LORENZO SEP 25-28
13 T MELISSA SEP 28-30
14 H NOEL OCT 28-NOV 2
15 T OLGA DEC 11-12
Hurricane (H)
Tropical Storm (T)
Tropical Depression
+-+-+ Extratropical
Wave/Low
Subtropical Depression
Subtropical Storm (ST)
Position at 0000 UTC
21 Position/date at 1200 UTC
5 Tropical Cyclone Number
Lambert Conformal Conic true at 20° and 40° North
North 0° South
5° West 0° East 5°

Hurricane Future Market

Market Quotes: DEAN

Date	Contract	Units	$Volume	LowPrice	HighPrice	AvgPrice	LastPrice
08/14/07	DEA-W	203	113.295	0.2	0.995	0.558	0.995

Modelisation of Financial Market Risks

- The finance academic Credo : markets are efficient

(They rationally aggregate information)

This implies certain types of stochastic processes underlying prices, in particular processes without memory (Markovian).

Typical examples : random walk, Ito process.

Mathematical tractability requires (Levy) stable laws (typical example : Gauss)

PART II : Risk Dependance

How to Model Risk Dependence

Variables : $(X_1,\ldots X_i,\ldots,X_n) = \mathbf{X}$

1) Deterministic models with noise

$$f(\mathbf{X}, \varepsilon) = 0$$

ε is a vector of (in general independent) random variables

Typical example: regression models

$X_1 = a + b_2X_2 + \ldots + b_nX_n + \varepsilon$ (Variable X_1 is privileged)

$X_1 = b_1.X_1 + b_2\mathrm{Log}(X_3) + b_4X_4^2 + b_5X_5.X_1 + \varepsilon$ (high flexibility for modelling)

2) Fully Stochatic models

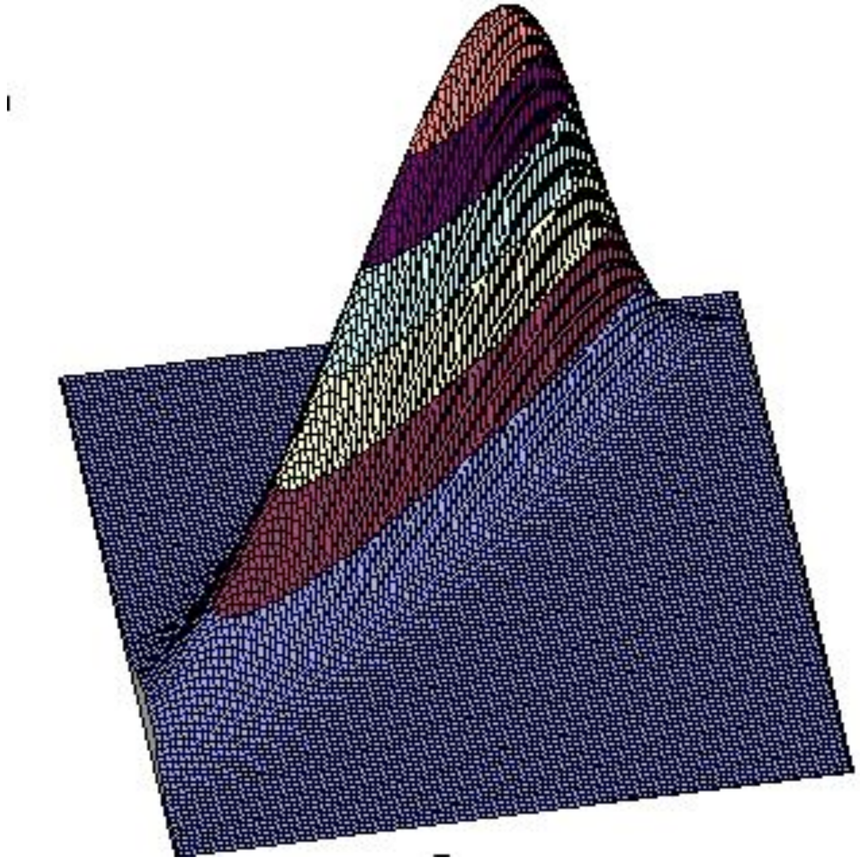
X is a random vector f(**X**) density F(**X**) (cumulative) probability

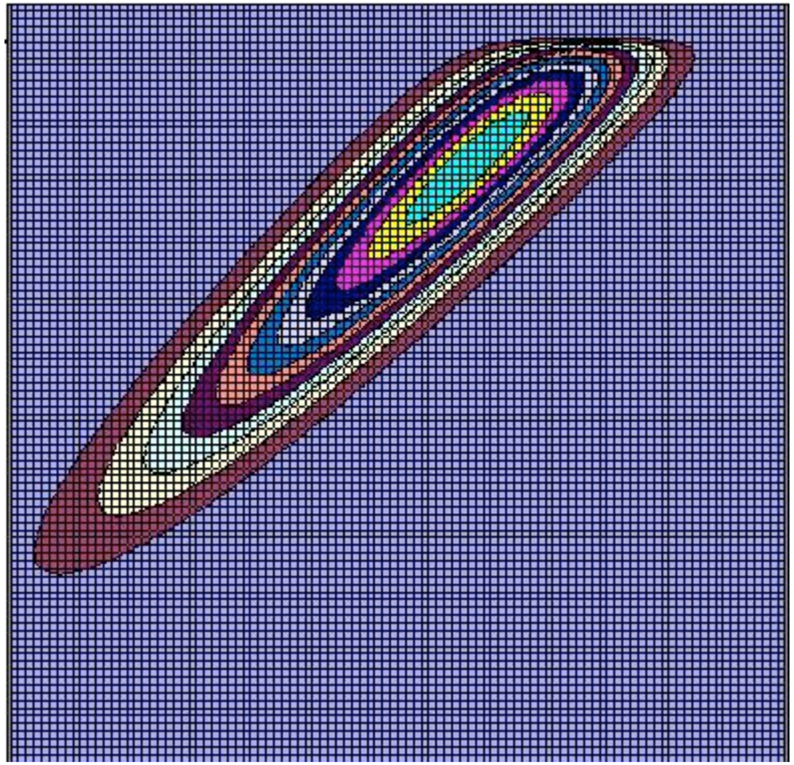
Typical examples :

X is normal with mean 0 and matrix variance Σ^2

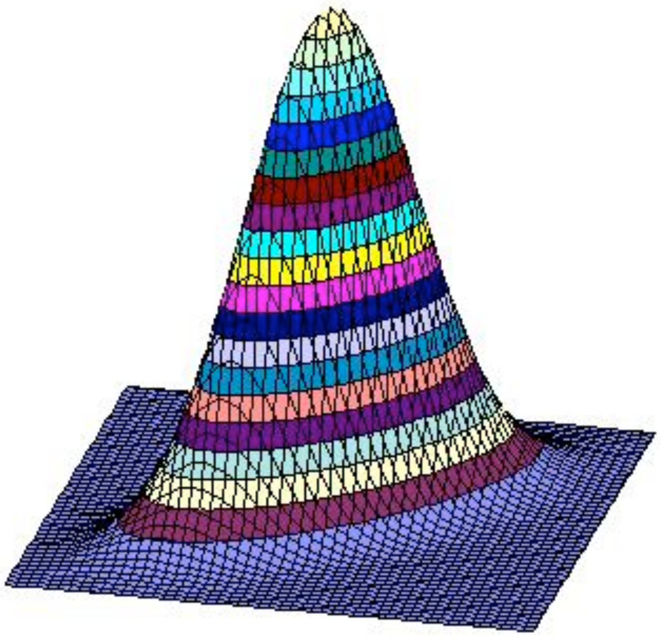
X is student with mean 0 and parameters Σ^2 and ν

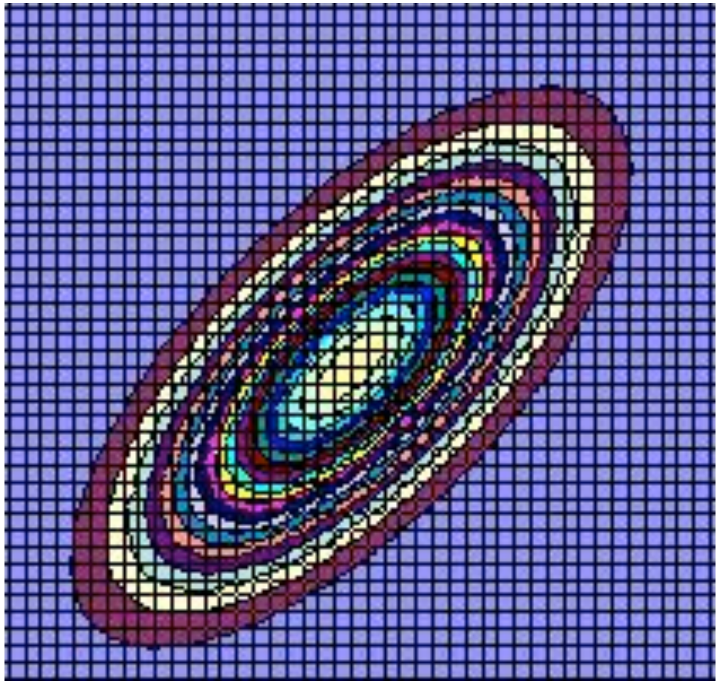
Model $Y = ax+b+\varepsilon$ x: beta and ε:normal





Gaussian Dependence





Elliptic dependence

Dependance and Marginal Distributions

Choosing a distribution leads to set jointly a *dependence* structure and a class of *marginal distributions.*

ex: **X** = (X_1,X_2) binormal implies:

- marginal X1 and marginal X2 normal
- elliptic dependence structure

Copulas : Separating Dependance and Marginals

(For the sake of simplicity : 2-dimensional variables)

A copulas is a continuous function $C(u_1,u_2)$, defined everywhere on the square $[0,1]^2$ such that:

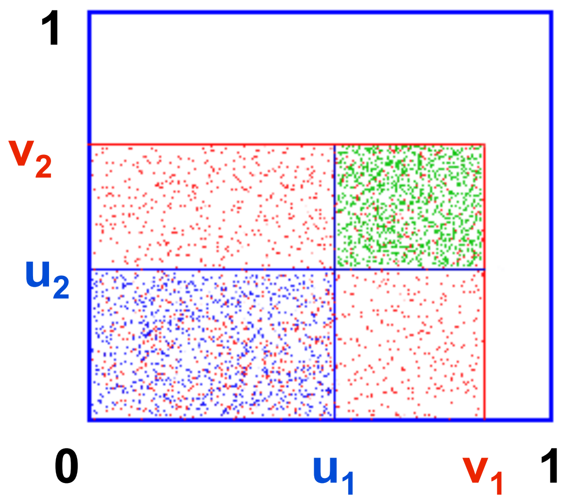
$C(0,0) = 0$

$C(u_1,0) = C(u_2,0)= 0$

$C(u_1,1)= u_1$ and $C(1,u_2)=u_2$ for any u_1 and u_2 in $[0,1]$

For $(v_1,v_2) >> (u_1,u_2)$:

$C(v_1,v_2)-C(v_1,u_2)-C(u_1,v_2)+C(u_1,u_2)\geq 0$



Sklar's Theorem

$F(x_1,x_2)$ bi-dimensional distribution

$F_1(x_1)$: continuous marginal distribution in x_1

$F_2(x_2)$: continuous marginal distribution in x_2

F can be written as:

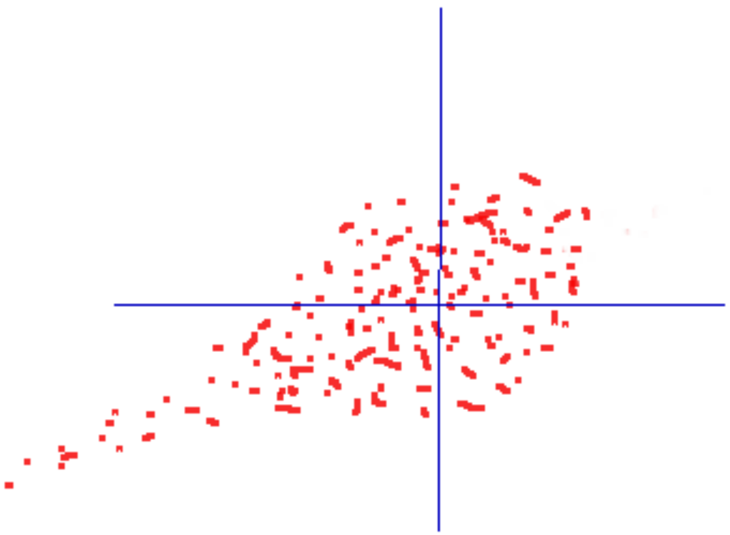
$$F(x_1,x_2) = C(F_1(x_1),F_2(x_2))$$

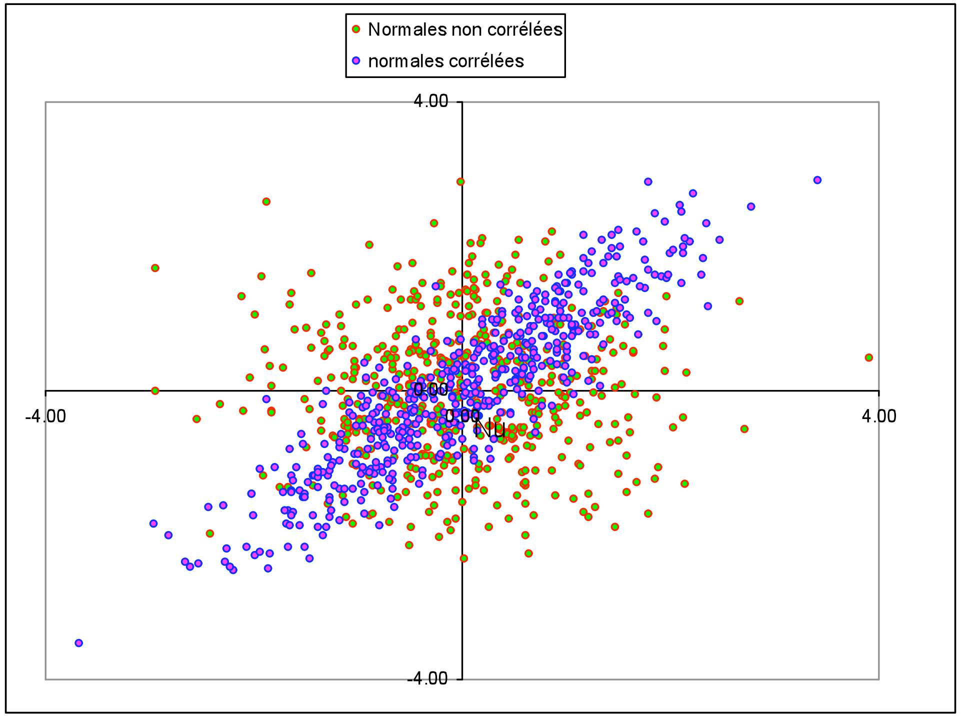
Where C is a (unique) copula

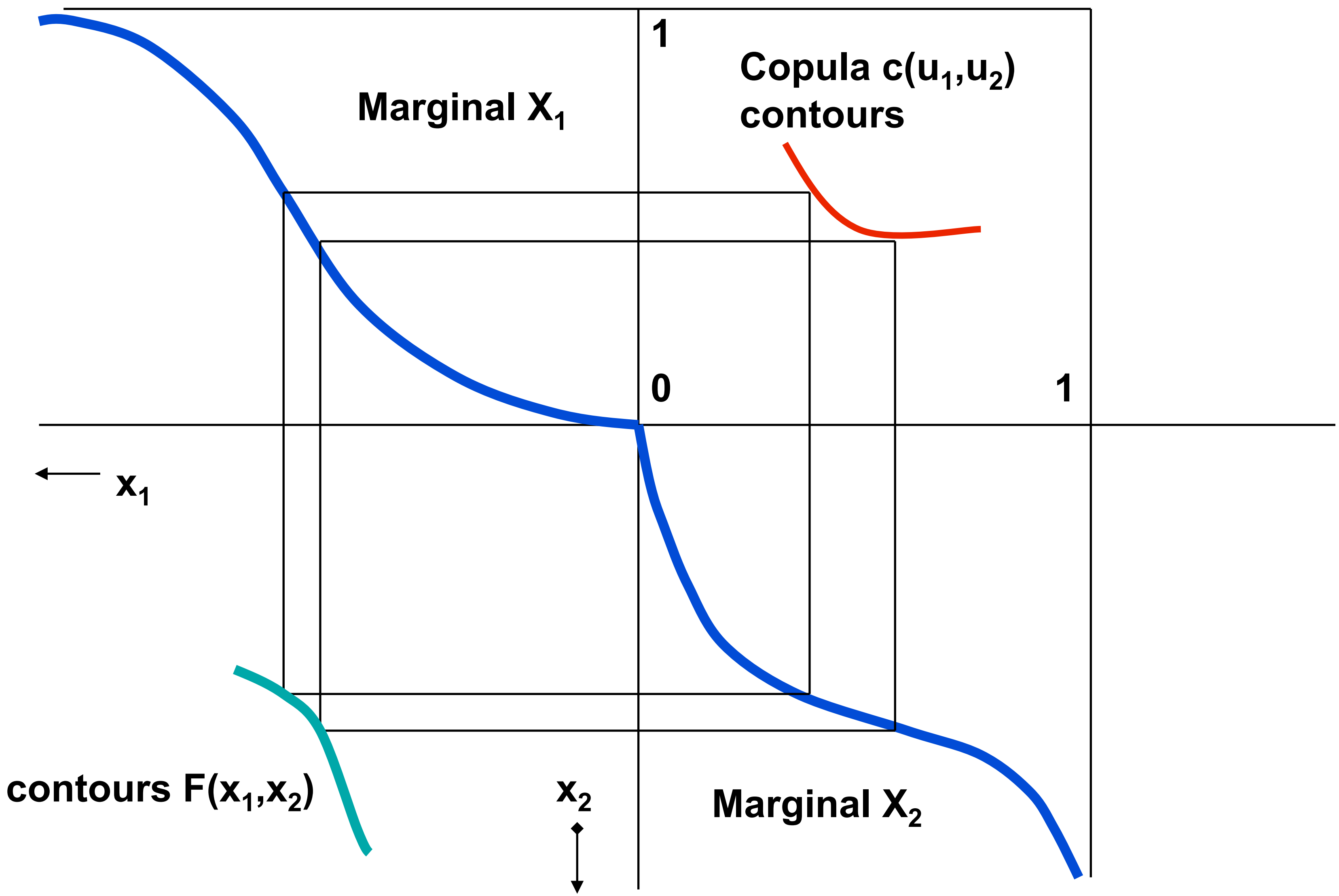
This theorem allows to "extract" the dependence structure from any distribution and use it with other marginals to reconstruct a new distribution.

Ex : Normal copula, Student copula,....

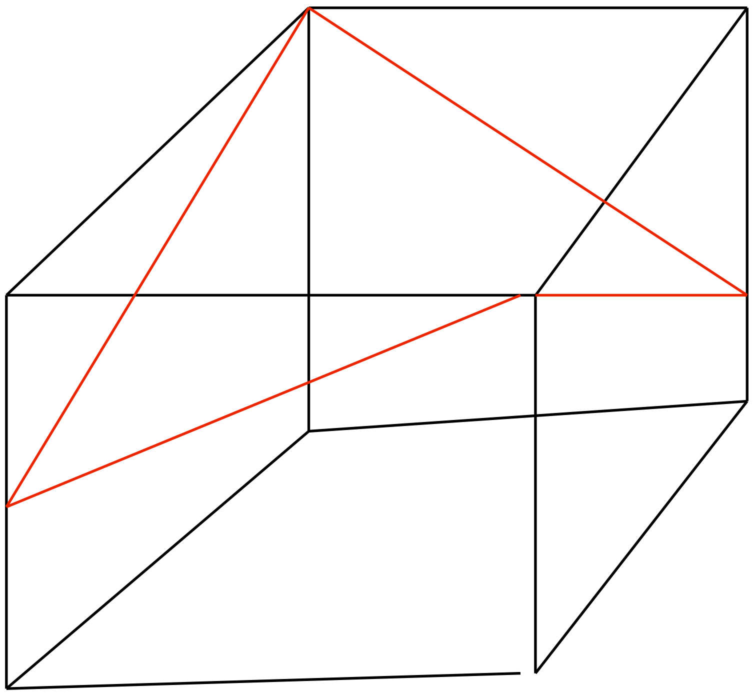
Gaussian and Non-Gaussian Dependence

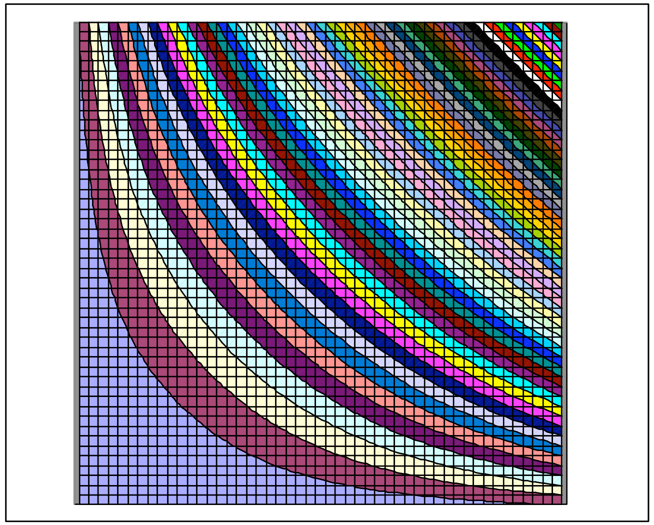


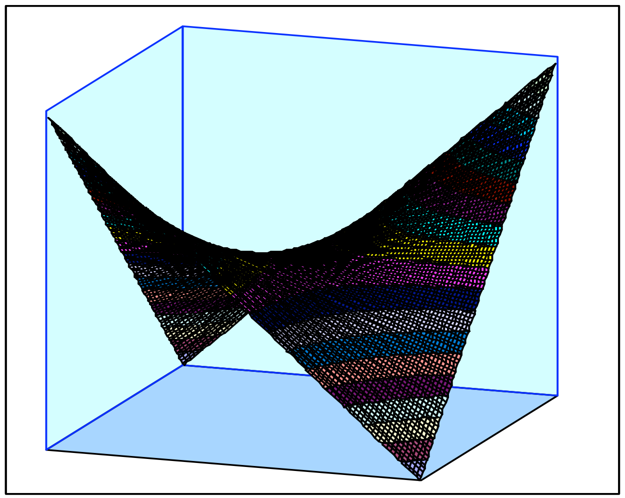


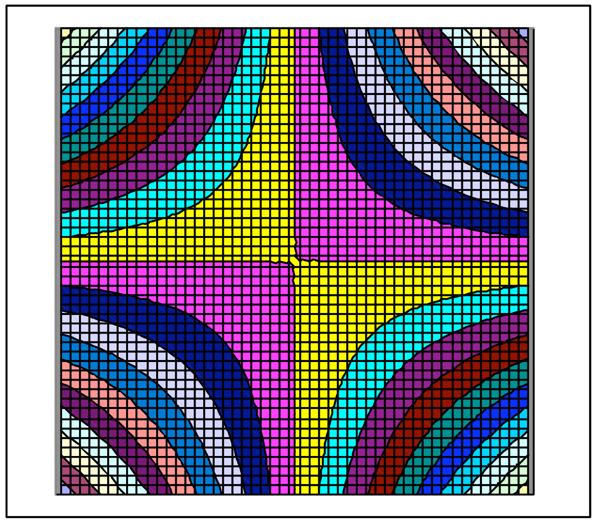

1
Copula $c(u_1,u_2)$
contours
Marginal X_1
0
1
x_1
contours $F(x_1,x_2)$
x_2
Marginal X_2

Copule linéaire

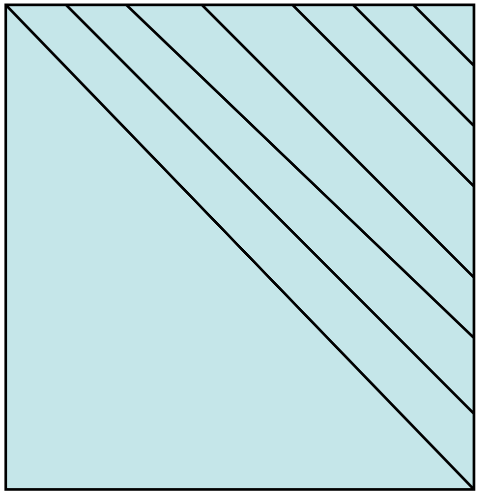






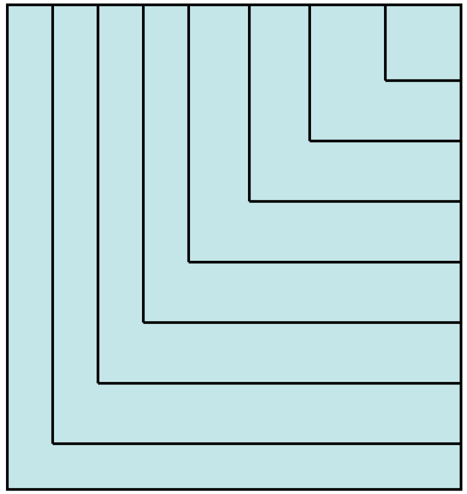


Frechet Bounds for Copulas



C^-

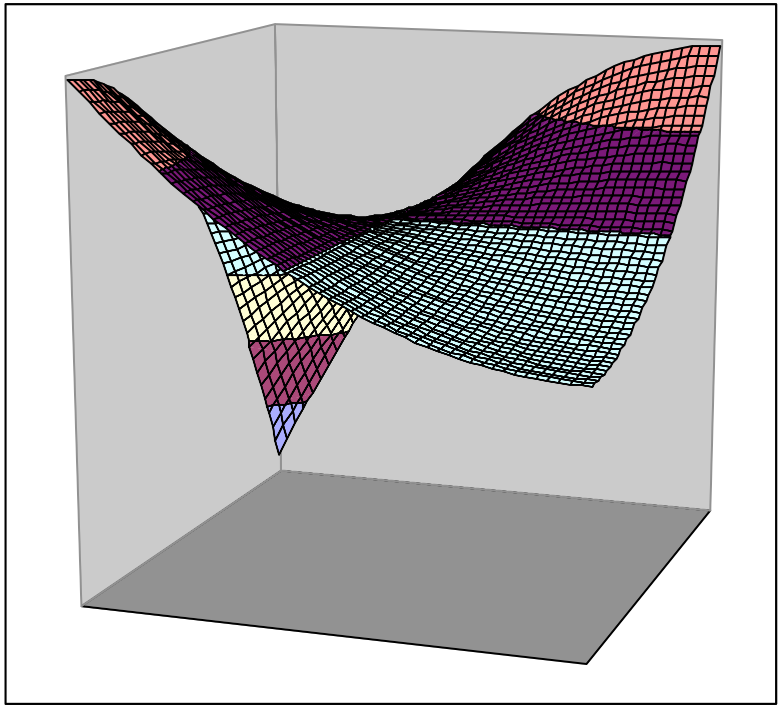
$\mathrm{Max}(u_1+u_2-1,0)$

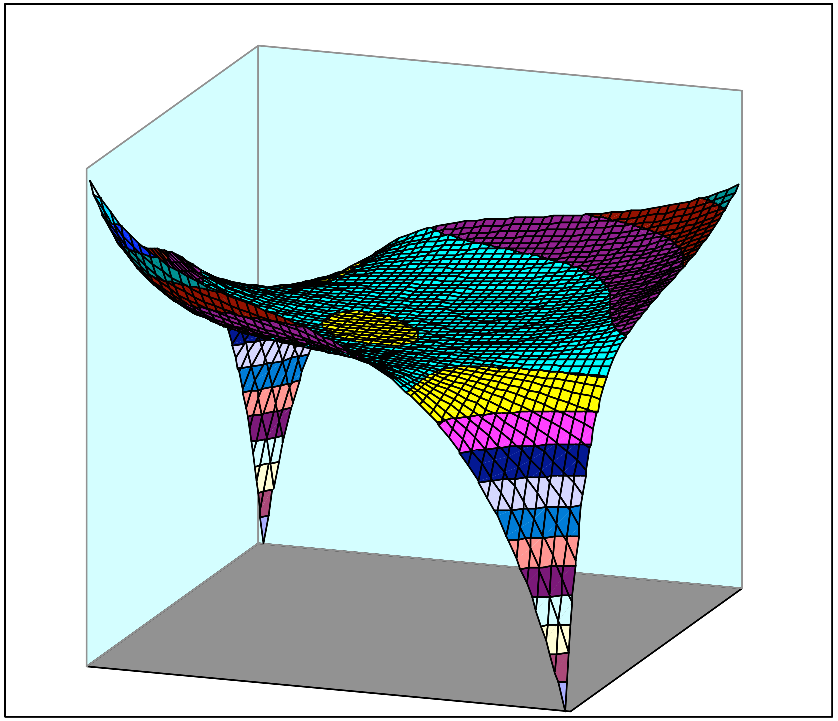


C^+

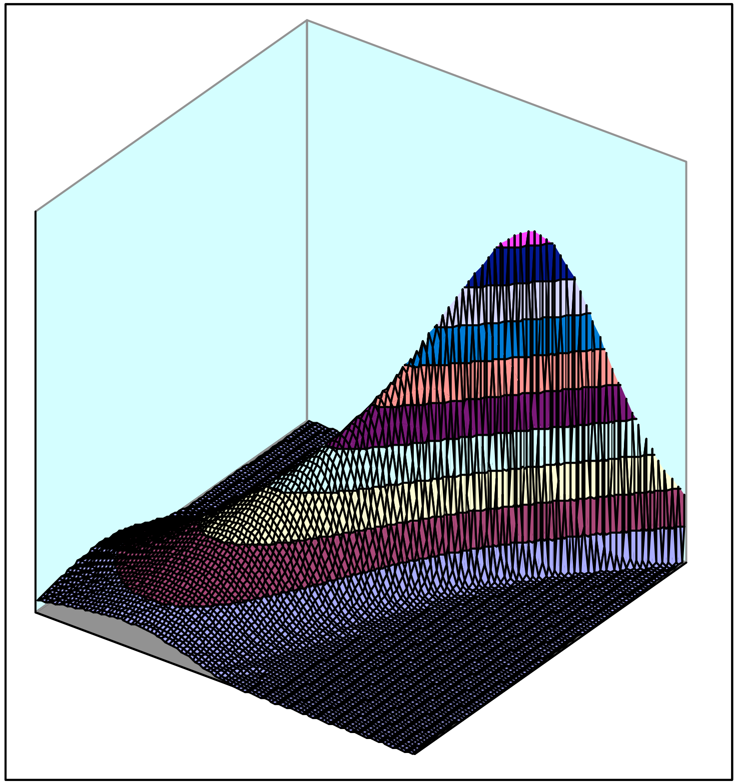
$\mathrm{Min}(u_1,u_2)$

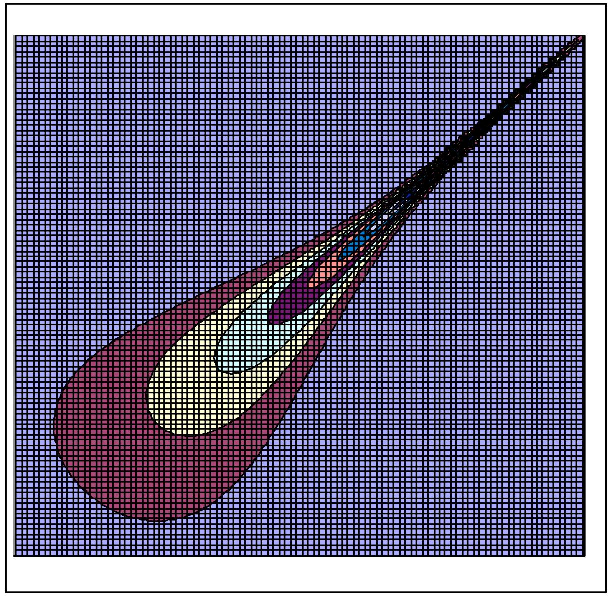
$$C^-(u_1,u_2) \leq C(u_1,u_2) \leq C^+(u_1,u_2)$$

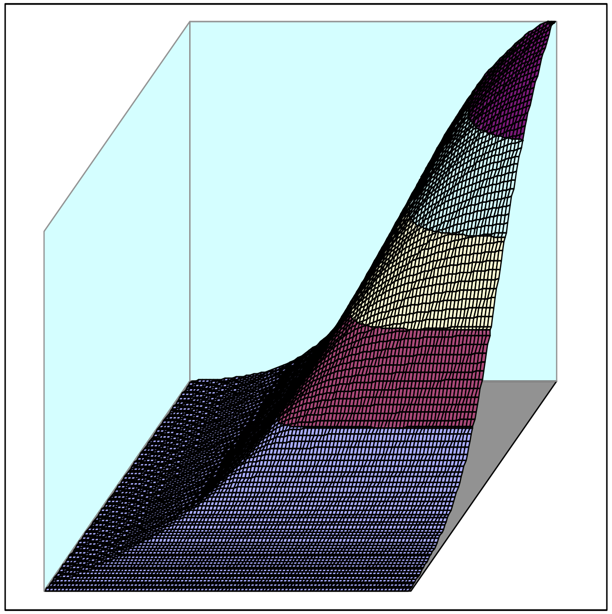




Clayton Copula $1/LOG_a(a^{1/x}+a^{1/y}-1)$

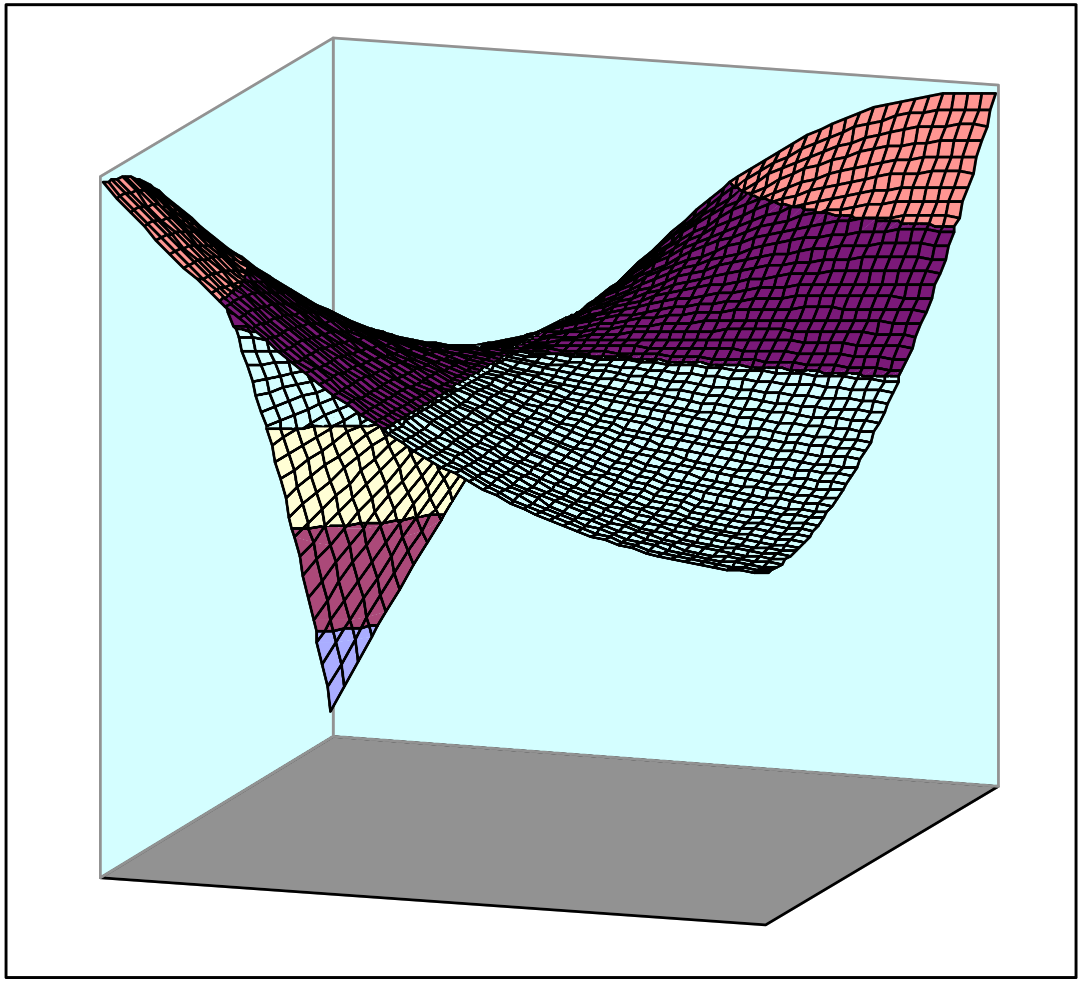




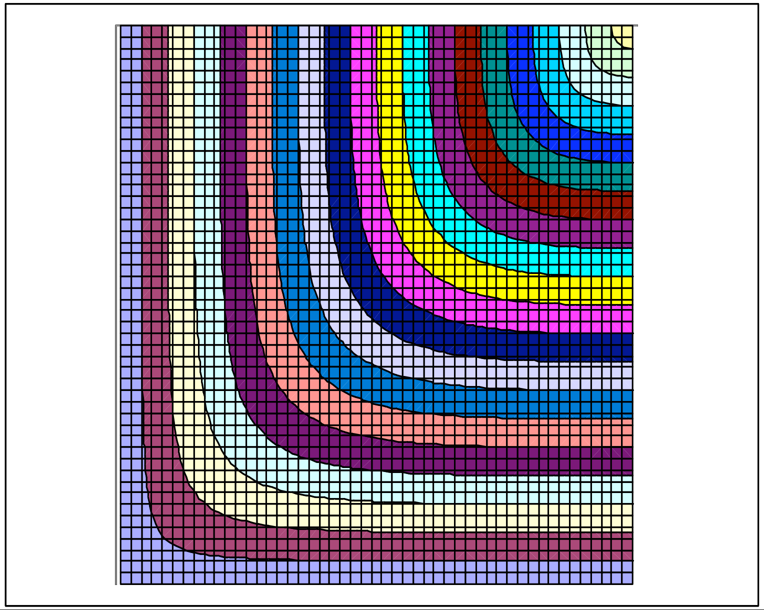


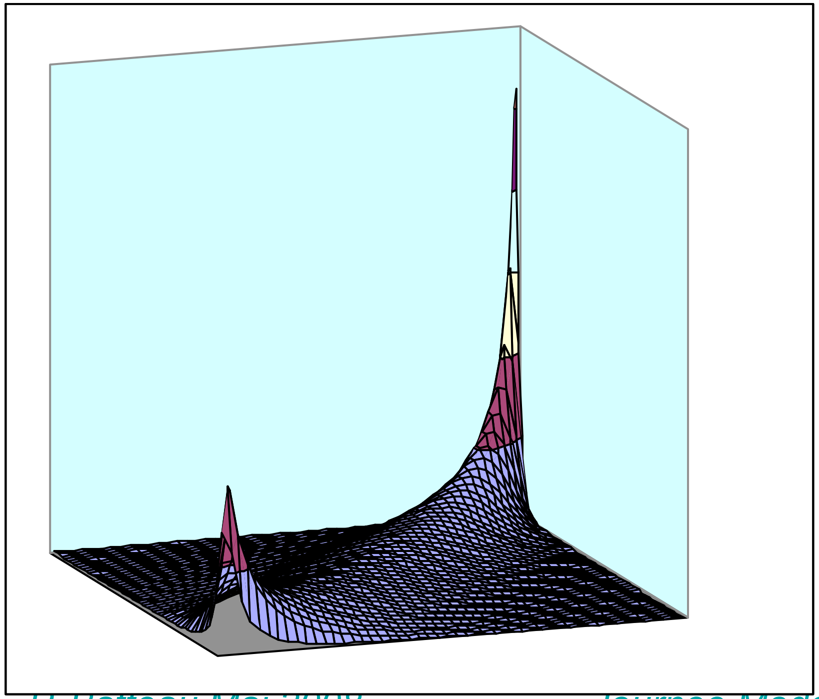
$EXP[-\gamma^* ((x-0.5)^2 + (y-0.5)^2 -(\alpha^*(x-y)^2+\beta^*(x+y-1))]$

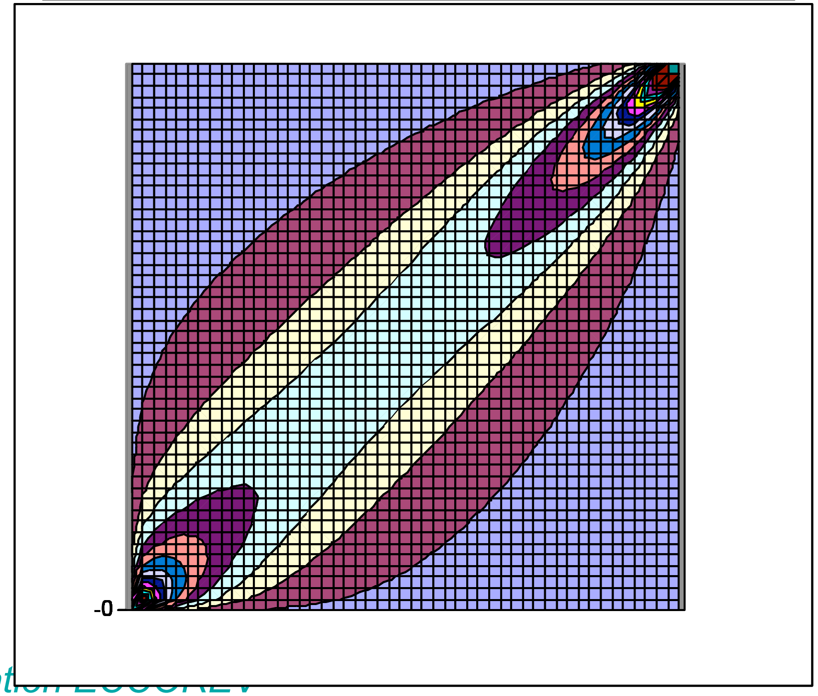
$\alpha = -3 \quad \beta = 2 \quad \gamma = 0.01$



Orthogonal copula
n odd
tends to Frechet's as n grows







Copula Estimation

General method : Maximum Likelihood

Copula limits :

Useful for spatial dependence

Difficulty to capture temporal dependence

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